First round
Dutch Mathematical Olympiad

16–27 January 2023

- Time available: 2 hours (120 minutes).
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact and simplified form, like $\frac{11}{81}$, $2+\frac{1}{2}\sqrt{5}$, $\frac{1}{4}\pi+1$, or 3^{100}.
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 28 January on the website: `www.wiskundeolympiade.nl`.
- Good luck!

A-problems

1. There is a power plant in the cell indicated by C. The cells indicated by X are places to which the power plant should be connected. This requires electricity masts in some cells, which we can indicate by placing an M in that cell. Two cells with a letter in them can be connected by a cable if the cells are adjacent at a side or a vertex. Ultimately, each cell with an X must be connected to the power plant via one or more cells with the letters M or X.

X		X		X
X				X
X		C		X

What is the minimum number of electricity masts needed?

A) 3 B) 4 C) 5 D) 8 E) 9

2. At the beginning of the day, a toy shop has 20 sticker sheets (0.30 euro each), 18 footballs (3 euros each), 5 teddy bears (5 euros each) and 8 water guns (15 euros each) in stock. The new cashier makes a mess of the records and reports at the end of the day about the sale of these four items only that the total amount is 75.80 euros, and that fewer footballs were sold than any other item.
How many teddy bears have been sold?

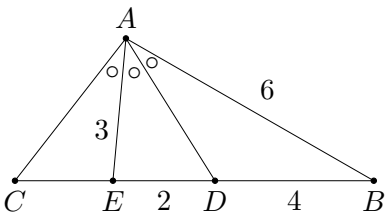
A) 1 B) 2 C) 3 D) 4 E) 5

3. A chess board consists of 8×8 squares. A knight is placed on a corner square. The knight makes jumps of either two squares horizontally and one square vertically, or two squares vertically and one square horizontally. On each square of the chess board, we write down how many jumps the knight needs to reach it.
Which number did we write down most often?

A) 2 B) 3 C) 4 D) 5 E) 6

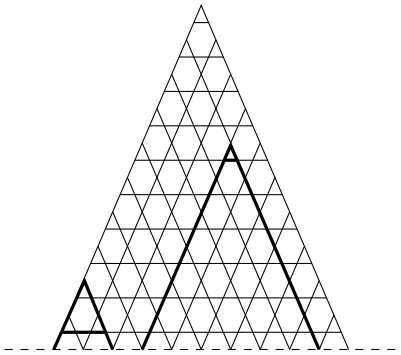
PLEASE CONTINUE ON THE OTHER SIDE

4. We have a triangle $\triangle ABC$ and two points D and E on the line segment BC, such that $\angle BAD = \angle DAE = \angle EAC$. Furthermore, $|AB| = 6$, $|BD| = 4$, $|DE| = 2$, and $|EA| = 3$. See the picture on the right: be aware, it is not drawn to scale!
What is the length of the line segment CE?



A) 2 B) $2\frac{1}{4}$ C) $2\frac{1}{2}$ D) $2\frac{3}{4}$ E) 3

5. In the figure on the right the capital letter A occurs very often. A capital letter A can be written in many ways: for two legs that meet at the top of the A, we can choose at which height we put the horizontal dash between the legs. The bottom of the legs of all the letters must be on the dotted line. Two of these capital letters A have been made bold in the figure: on the left, an A with the dash quite low, on the right, an A with the dash very high.
How many capital letters A can be found in this figure?



A) 55 B) 110 C) 220 D) 275 E) 550

6. There are 2023 people sitting at a round table. Each person is either a knave or a knight. Knights always speak the truth and knaves always lie. The first person says: "There is at least one knave at this table." The person to the left says: "There is at least one knight at this table." The third one says: "There are at least two knaves at this table." The fourth says: "There are at least two knights at this table." This continues, until the last person at the table says: "There are at least 1012 knaves at this table." The first person, who has already made a statement, now says: "There are at least 1012 knights at this table."
How many knaves are sitting at the table?

A) 505 B) 506 C) 1011 D) 1012 E) 1507

7. How many pairs of positive integers a and b with $a < b$ are there such that

$$\frac{1}{a} + \frac{1}{b} = \frac{4}{15}?$$

A) 1 B) 2 C) 3 D) 4 E) 5

8. Anna, Bella, and Celine are going to colour. There are six pencils, with different colours. They divide the pencils among themselves, in such a way that everyone gets at least one pencil.
In how many different ways can they distribute the pencils?

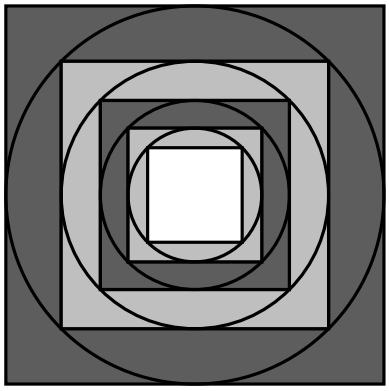
A) 537 B) 540 C) 720 D) 729 E) 1170

B-problems

1. Albert makes a sequence of numbers, the first 2023 of which are exactly the numbers 1 through 2023, in some order. To determine each subsequent number, Albert takes the *median* of the 2023 previous numbers. You find the median of 2023 numbers by sorting them by size and taking exactly the middle number from that sequence.
How many different values can the three thousandth number of Albert's sequence have?

2. For a three-digit positive integer, we can multiply the three digits together. We call the result the *digit product* of that number. For example, 123 has digit product $1 \times 2 \times 3 = 6$ and 524 has digit product $5 \times 2 \times 4 = 40$. A number cannot begin with the digit 0.
Determine the three-digit number that equals exactly five times its own digit product.

3. In a square, we draw the inscribed circle, into which we draw an inscribed square whose sides are parallel to the sides of the original square. We colour the area between these squares dark grey. In the smaller square, we again draw the inscribed circle containing another inscribed square with parallel sides; the area between these we colour light grey. We repeat this process until there are 2023 dark grey and 2023 light grey areas. In the picture, you can see the first steps drawn. The inner square remains white. The area of all the dark grey areas together is exactly 1.
What is the area of all the light grey areas together?



4. A positive integer a consists of four digits, three of which are equal to each other. The square of a consists of seven digits, all of which are different. The number b arises by reading number a from right to left. It turns out that b is greater than a. In addition, it turns out that b^2 is exactly equal to a^2 read from right to left.
Find all possibilities for a.